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FAULHABER Tutorial

Motor Calculations for Coreless Brush DC Motors

WE CREATE MOTION

An empirical and theoretical approach

When selecting a Coreless Brush DC Motor for an application, or when developing a powered prototype, there are several basic motor physics principles which must be considered to produce a safe, well-functioning, sufficiently-powered precision drive system. In this document, we have provided some important methods, formulas and calculation details to determine the power output of a coreless motor, the speed-torque curve of the motor, the current and efficiency plots, and the theoretical cold calculations that estimate motor performance.

For additional assistance, contact us to speak with an application engineer.

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1. Calculating Initial Power Requirements

DC motors are transducers because they convert electrical power (P_{in}) into mechanical power (P_{out}). The quotient of both terms equates to the efficiency of the motor. The frictional losses and copper losses result in total power loss (P_{loss}) in Joules/sec (Iron losses in coreless DC motors are negligible). There are additional losses due to heat rise, but we will discuss those below:

$$P_{in} = P_{out} + P_{loss}$$

In physics, power is defined as the rate of doing work. The standard metric unit of power is the "Watt" W. How is power calculated? For linear motion, power is the product of force and distance per unit time $P = F \cdot (d/t)$. Since speed is distance over time, the equation becomes $P = F \cdot s$. In the case of rotational motion, the analogous calculation for power is the product of torque and angular distance per unit time or simply the product of torque and angular speed.

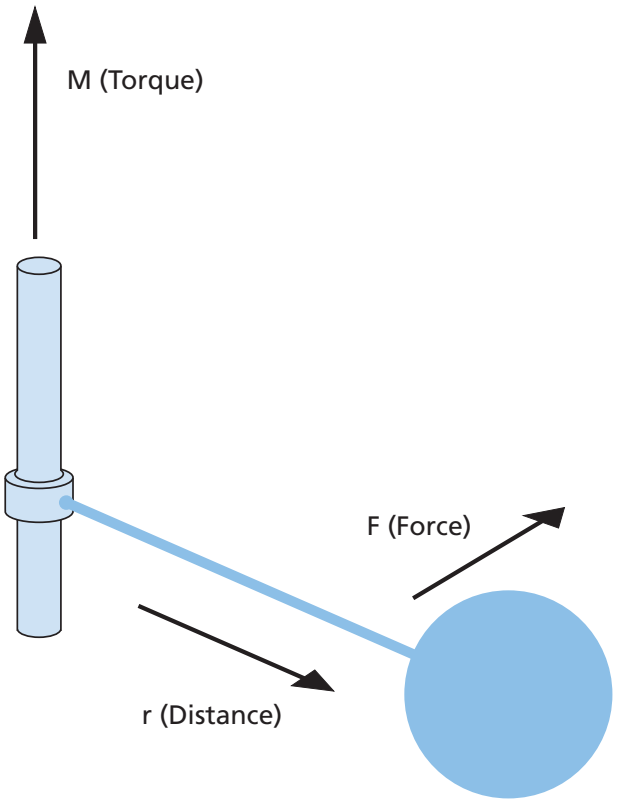
$$P_{out} = M \cdot \omega_{rad}$$

Where:

P = Power in W
M = Torque in Nm
F = Force in N
d = Distance in m
t = Time in s
ω_{rad} = Angular Velocity in rad/s

The symbol used for torque is normally the lower-case Greek letter "τ"(tau) or sometimes just the letter "T". However, when it is called Moment of Force, it is commonly denoted by the letter "M".

European nomenclature often employs the lowercase letter "n" to represent speed about an axis. Usually "n" is in units of revolutions per minute or rpm.



It is important to consider units when calculating mechanical power. In calculating power, if "n" (speed) is in **min**$^{-1}$, then you must convert it to angular velocity in units of **rad/s**. This is accomplished by multiplying the speed by a unit conversion factor of **2π/60**. In addition, if "M" (torque) is in **mNm**, then we must multiply it by **10**$^{-3}$ (divide by 1 000) to convert the units to **Nm** for calculation purposes.

$$\omega_{rad} = n \cdot \left(\frac{2\pi}{60}\right)$$

$$P_{out} = M \cdot (10^{-3}) \cdot n \cdot \left(\frac{2\pi}{60}\right)$$

Where:

n = Speed in min^{-1}
M = Torque in mNm

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DC Micromotor 2668 CR

Assume that it is necessary to determine the power of which a particular motor, 2668W024CR is expected to deliver at cold operation with a torque load of 68 mNm at a speed of 7 370 min^{-1}. The product of the torque, speed, and the appropriate conversion factor is shown below.

$$P_{mech} = M \cdot n \cdot \left(\frac{2\pi}{60.000}\right)$$

$$P_{mech} = 68\ \text{mNm} \cdot 7370\ \text{rpm} \cdot \left(\frac{2\pi}{60.000}\right) = 52\ \text{W}$$

Calculation of initial power requirements is often used as a preliminary step in motor or gear motor selection. If the mechanical output power required for a given application is known, then the maximum or continuous power ratings for various motors can be examined to determine which motors are possible candidates for use in the application.

2. Finding Motor Parameters Empirically

We've provided a method below to determine the motor parameters using the 2668W024CR coreless DC motor as an example. First we'll explain a more empirical approach, then we'll perform a theoretical calculation.

One commonly used method of plotting motor characteristics graphically is the use of torque-speed curves. While the use of torque-speed curves is much more common in technical literature for larger DC machines than it is for small, ironless core devices, the technique is applicable in either case.

Normally, torque-speed curves are generated by plotting motor speed, motor current, mechanical output power, and efficiency as functions of the motor torque. The following discussion will describe the construction of a set of torque-speed curves for a typical DC motor from a series of raw data measurements.

The 2668W024CR has a nominal voltage of 24 V.
If you have a few fundamental pieces of laboratory equipment, you can measure the torque-speed curves for a 2668 CR series coreless dc motor at a specified operating point.

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Step 1: Measure Basic Parameters

Many parameters can be obtained directly by using a motion controller, such as one of the FAULHABER MC3 motion controllers. Most controller manufacturers offer software, such as the FAULHABER Motion Manager, which includes a trace recording function that plots voltage, current, position, speed, etc. They can also provide an accurate snapshot of motor operation in great detail. For example, the MC3 family of motion controllers (the MC 5004, MC 5005 and MC 5010) all can measure a plethora of motion parameters. This is probably the quickest method for obtaining the data to plot a torque-speed curve, but it is not the only method.

If a controller with trace recording capability is not available, then we can also use some basic lab equipment to characterize a motor under stall, nominal and no load conditions. Using a voltage supply set to 24 V, run the 2668W024CR unloaded and measure the rotational speed using a non-contacting tachometer (a strobe, for instance). Also, measure the motor current under this no-load condition. A current probe is ideal for this measurement since it does not add resistance in series with the operating motor. Using an adjustable torque load such as a small particle brake or an adjustable hysteresis dynamometer, a load can be coupled to the motor shaft.

Now increase the torque to the motor just to the point where stall occurs. At stall, measure the torque from the brake and the motor current. For the sake of this discussion, assume that the coupling adds no load to the motor and that the load from the brake does not include unknown frictional components. It is also useful at this point to measure the terminal resistance of the motor. Measure the resistance by contacting the motor terminals with an ohmmeter. Then spin the motor shaft and take another measurement. The measurements should be very close in value. Continue to spin the shaft and take at least three measurements. This will ensure that the measurements were not taken at a point of minimum contact on the commutator.

Now we have measured the:

n_0 = No-Load speed
I_0 = No-Load current
M_H = Stall Torque
R = Terminal Resistance

Step 2: Plot Current vs. Torque and Speed vs. Torque

You may prepare a graph with motor torque on the abscissa (horizontal axis), speed on the left-side ordinate (vertical axis), and current on the right-side ordinate. Scale the axes based on the measurements you took in step one. Draw a straight line from the left origin of the graph (zero torque and zero current) to the stall current on the right-side ordinate (stall torque and stall current). This line represents a plot of the motor current as a function of the motor torque. The slope of this line is the current constant k_I which is the proportionality constant for the relationship between motor current and motor torque (in units of current per unit torque or A/mNm). The reciprocal of this slope is the torque constant k_M (in units of torque per unit current or mNm/A).

$$k_I = \frac{1}{k_M}$$

Where:

k_I = Current constant
k_M = Torque constant

For the purpose of this discussion, it will be assumed that the motor has no internal friction. In practice, the motor friction torque M_R is determined multiplying the torque constant k_M of the motor and the measured no-load current I_0. The torque vs. speed line and the torque vs. current line are then started not at the left vertical axis, but at an offset on the horizontal axis equal to the calculated friction torque.

$$M_R = k_M \cdot I_0$$

Where:

M_R = Friction Torque

Step 3: Plot Power vs. Torque and Efficiency vs. Torque

In most cases, two additional vertical axes can be added for plotting power and efficiency as functions of torque. A second vertical axis is usually used for efficiency and a third vertical axis can be used for power. For the sake of simplifying this discussion, efficiency vs. torque and power vs. torque will be plotted on the same graph as the speed vs. torque and current vs. torque plots (example shown below).

Construct a table of the motor mechanical power at various points from no-load to stall torque. Since mechanical power output is simply the product of torque and speed with a correction factor for units (see section on calculating initial power requirements), power can be calculated using the previously plotted line for speed vs. torque.

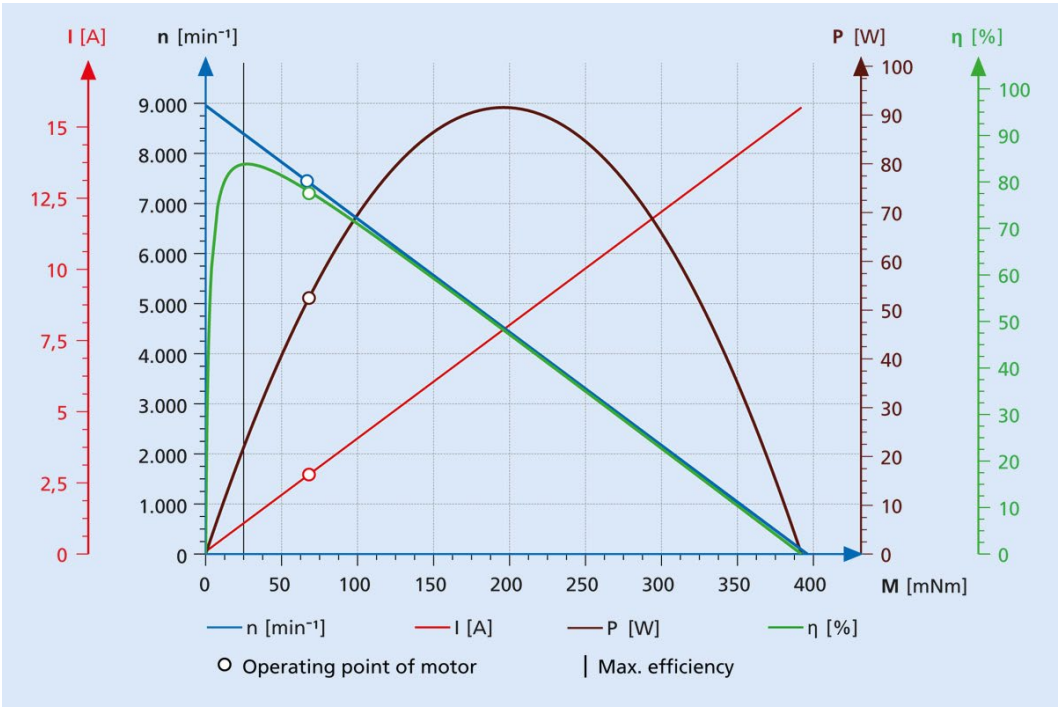
A sample table of calculations for motor 2668W024CR is shown in Table 1. Each calculated point for power is then plotted. The resulting function is a parabolic curve as shown below in Graph 1. The maximum mechanical power occurs at approximately one-half of the stall torque. The speed at this point is approximately one-half of the no-load speed.

Construct a table on a spreadsheet of the motor efficiency at various points from no-load speed to the stall torque. The voltage applied to the motor is given, and the current at various levels of torque has been plotted. The product of the motor current and the applied voltage is the power input to the motor. At each point selected for calculation, the efficiency η of the motor is the mechanical power output divided by the electrical power input. Again, a sample table for motor 2668W024CR is shown in Table 1 and a sample curve in Graph 1. Maximum efficiency occurs at about 10% of the motor stall torque.

Plot definitions

- Blue = Speed vs. Torque (*n* vs. *M*)
- Red = Current vs. Torque (*I* vs. *M*)
- Green = Efficiency vs. Torque (η vs. *M*)
- Brown = Power vs. Torque (*P* vs. *M*)

Characteristic curves of Motor



Note: Note how all four solid plots change as a result of increased resistance in the copper windings and weakened torque output, due to heat rise. So your results may differ slightly depending on whether your motor is cold or warm when you plot your graphs.

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Results of the Load Calculation

Load current	**2,79**	A
Load voltage	**24,11**	V
Motor winding temperature	**140,23**	°C
Motor housing temperature	**105,03**	°C
Calculated speed (motor)	**7.370**	min^{-1}
Required load torque	**68**	mNm
Output power	**52,49**	W
Efficiency (over all)	**77,97**	%

Overall Dimensions

Diameter	**26**	mm
Length	**68**	mm
Mass	**189**	g

Note: Due to space constraints, only an example calculation for one point is shown.

Calculating Motor Parameters Theoretically

Another useful parameter in motor sizing is the motor constant. Utilizing this figure of merit properly will substantially reduce the iterative process in selecting a DC motor. It simply measures a transducer's intrinsic ability to transform electrical power to mechanical.

$$k_m = \frac{M}{\sqrt{(P_{in} - P_{out})}}$$

Maximum efficiency occurs at about 10% of the motor stall torque. The denominator is known as the resistive power loss. With some algebraic manipulation, the equation can be simplified to:

$$k_m = \frac{k_M}{\sqrt{R}}$$

Please keep in mind that k_m (motor constant), shouldn't be confused with k_M (torque constant). Note that the motor constant subscript is a lowercase "*m*" while the torque constant subscript utilizes an uppercase "*M*".

For a brush or brushless DC motor of relatively small size, the relationships that govern the behavior of the motor in various circumstances can be derived from the laws of physics and the characteristics of the motors themselves. Kirchoff's voltage rule states, "The sum of the potential increases in a circuit loop must equal the sum of the potential decreases." When applied to a DC motor connected in series with a DC power source, Kirchoff's voltage rule can be expressed as "The nominal supply voltage from the power source must be equal in magnitude to the sum of the voltage drop across the resistance of the windings and the Back-EMF generated by the motor."

$$U = I \cdot R + U_E$$

Where:

U = Power supply in V
I = Current in A
R = Terminal resistance in Ω
U_E = Back-EMF in V

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The Back-EMF generated by the motor is directly proportional to the angular velocity of the motor. The proportionality constant is the Back-EMF constant of the motor.

$$U_E = k_E \cdot \omega$$

Where:
ω = Angular velocity of the motor
k_E = Back-EMF constant of the motor

Therefore, by substitution:

$$U = I \cdot R + k_E \cdot \omega$$

The Back-EMF constant of the motor is usually specified by the motor manufacturer in V/rpm or mV/rpm. In order to arrive at a meaningful value for the Back-EMF, it is necessary to specify the motor velocity in units compatible with the specified Back-EMF constant.

"The sum of the potential increases in a circuit loop must equal the sum of the potential decreases."
(Kirchoff's voltage rule)

The motor constant is a function of the coil design and the strength and direction of the flux lines in the air gap. Although it can be shown that the three motor constants normally specified (back EMF constant, torque constant, and speed constant) are equal if the proper units are used, calculation is facilitated by the specification of three constants in the commonly accepted units.

The torque produced by the rotor is directly proportional to the current in the armature windings. The proportionality constant is the torque constant of the motor.

$$M_m = I \cdot k_M$$
$$\frac{M_m}{k_M} = I$$

Where:
M_m = Torque developed at motor
k_M = Motor torque constant

Substituting this ratio for current yields:

$$U = \left(\frac{M_m}{k_M}\right) \cdot R + k_E \cdot \omega$$

The torque developed at the rotor is equal to the friction torque of the motor plus the load torque (due to external mechanical loading):

$$M_m = M_L + M_R$$

Where:
M_R = Motor friction torque
M_L = Load torque

Assuming that a constant voltage is applied to the motor terminals, the motor velocity will be directly proportional to sum of the friction torque and the load torque. The constant of proportionality is the slope of the torque-speed curve. Motor performance is better when this slope is a smaller number. The steeper the drop in slope, the poorer the performance one can expect from a given coreless motor. This ratio can be calculated by:

$$\frac{\Delta n}{\Delta M} = \frac{n_0}{M_H}$$

Where:
Δn = Change in speed
ΔM = Change in torque
M_H = Stall torque
n_0 = No-Load speed

An alternative approach to deriving this value is to solve for speed, n:

$$n = \frac{U}{k_E} - \frac{M}{(k_m \cdot k_E)}$$

Using Calculus, we differentiate both sides with respect to M, which yields:

$$\frac{d_n}{d_M} = \frac{R}{(k_m \cdot k_E)}$$

or

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$$\frac{\Delta n}{\Delta M} = \frac{R}{\left(k_M^2 \cdot \frac{2\pi}{60.000}\right)}$$

Although we are not showing a negative sign here, it is implied that the result will yield a declining (negative) slope.

Theoretical Motor Calculation Example

Let's delve a bit into the theoretical calculations. The 2668W024CR coreless DC motor is to be operated with 24 V applied to the motor terminals and a torque load of 68 mNm. Find the resulting motor constant, motor speed, motor current, motor efficiency, and power output. From the motor datasheet, it can be seen that the no-load speed of the motor at 24 V is 7800 min^{-1}. If a torque load is not coupled to the motor shaft, the motor would run at this speed.

First, let's get a general idea of motor performance by calculating the motor constant k_m. In this case, we arrive at a constant of 28,48 mNm/A. According to the data sheet of the motor, the electrical resistance is 1.03 Ohm in the cold state for the 24V variant.

$$k_m = \frac{k_M}{\sqrt{R}}$$

$$k_m = \frac{28{,}9}{\sqrt{1{,}03}} = 28{,}48 \text{ mNm/A}$$

The motor speed under load is simply the no-load speed less the reduction in speed due to the load. The proportionality constant for the relationship between motor speed and motor torque is the slope of the torque vs. speed curve, given by the motor no-load speed divided by the stall torque. In this example, we'll calculate the speed reduction (neglecting temperature effects) caused by the 68 mNm torque load by cancelling out the mNm units:

$$\frac{\Delta n}{\Delta M} = \frac{R}{\left(k_M^2 \cdot \frac{2\pi}{60.000}\right)}$$

$$\frac{\Delta n}{\Delta M} = \frac{1{,}03\ \Omega}{28{,}9^2 \text{ mNm/A} \cdot \left(\frac{\pi}{30.000}\right)} = 11{,}8 \text{ min}^{-1}\text{/mNm}$$

Now through substitution:

$$n_L = M\left(\frac{\Delta n}{\Delta M}\right)$$

$$n_L = 68 \text{ mNm} \cdot \left(\frac{11{,}8 \text{ min}^{-1}}{\text{mNm}}\right) = 802 \text{ min}^{-1}$$

The motor speed under load must then be approximately:

$$n = 7800 \text{ min}^{-1} - 802 \text{ min}^{-1} = 6998 \text{ min}^{-1}$$

The motor current under load is the sum of the no-load current and the current resulting from the load.

The proportionality constant relating current to torque load is the torque constant (k_M). That value is 28.9 mNm/A. By taking the reciprocal, we end up with the current constant k_I which can help us calculate the current at load. In this case, the load is 68 mNm and the current resulting from that load (neglecting heat rise) equates to approximately:

$$I_{Load} = M \cdot k_I$$

$$I_{Load} = 68 \text{ mNm} \cdot 0{,}035 \text{ A/mNm} \approx 2{,}380 \text{ A}$$

The total motor current can be approximated by summing this value with the motor no-load current. The datasheet lists the motor no-load current as 78 mA. After rounding, the total current is roughly:

$$I = I_{Load} + I_0$$

$$I = 2{,}380 \text{ A} + 0{,}078 \text{ A} \approx 2{,}458 \text{ A}$$

The mechanical power output of the motor is simply the product of the motor speed and the torque load with a correction factor for units (if required). Therefore, the output power of the motor would be approximately:

$$P_{out} = M \cdot n \cdot \left(\frac{2\pi}{60.000}\right)$$

$$P_{out} = 68 \text{ mNm} \cdot 6998 \text{ rpm} \cdot \left(\frac{2\pi}{60.000}\right) \approx 49{,}83 \text{ W}$$

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The electrical power input to the motor is the product of the applied voltage and the total motor current in Amps. In this application:

$$P_{in} = U \cdot I$$

$$P_{in} = 24\ \text{VDC} \cdot 2{,}458\ \text{A} = 58{,}99\ \text{W}$$

Since the efficiency η is simply power out divided by power in, let's calculate it at our operating point:

$$\eta = \frac{P_{out}}{P_{in}}$$

$$\eta = \frac{49{,}83\ \text{W}}{58{,}99\ \text{W}} = 0{,}844 = 84\%$$

Estimating Motor Winding Temperature During Operation:

A current I flowing through a resistance R results in a power loss as heat of $I^2 \cdot R$. In the case of a DC motor, the product of the square of the total motor current and the armature resistance is the power loss as heat in the armature windings. For example, if the total motor current was 0,203 A and the armature resistance 14,5 Ω the power lost as heat in the windings is:

$$P_{loss} = I^2 \cdot R$$

$$P_{loss} = (2{,}458\ \text{A})^2 \cdot 1{,}03\ \Omega = 6{,}22\ \text{W}$$

The heat resulting from $I^2 \cdot R$ losses in the coil is dissipated by thermal conduction through motor components and airflow in the air gap. The ease with which this heat can be dissipated in a motor (or any system) is defined by thermal resistance.

Thermal resistance (which is the reciprocal of thermal conductance) represents how well a material resists heat transfer through a defined path. Motor manufacturers typically provide an indication of the motor's ability to dissipate heat by providing thermal resistance R_{th} values. A large cross section aluminum plate would have a very low thermal resistance, for example, while the values for air or a vacuum would be considerably higher. In the case of DC motors, there is a thermal path from the motor windings to the motor case and a second thermal pass between the motor case and the motor environment (ambient air, etc.). Some motor manufacturers specify a thermal resistance for each of the two thermal paths while others specify only the sum of the two as the total thermal resistance of the motor. Thermal resistance values are specified in temperature increase per unit power loss. The total $I^2 \cdot R$ losses in the coil (the heat source) are multiplied by thermal resistances to determine the steady state armature temperature. The steady state temperature increase of the motor (T) is given by:

$$\Delta T = I^2 \cdot R \cdot (R_{th1} + R_{th2})$$

Where:

ΔT = Change in Temperature in K
I = Current through motor windings in A
R = Resistance of motor windings in Ω
R_{th1} = Thermal resistance from windings to case in K/W
R_{th2} = Thermal resistance case to ambient in K/W

Let's continue our example using the 2668W024CR motor running with a current of 2,458 A in the motor windings, with an armature resistance of 1,03 Ω, a winding-to-case thermal resistance of 3 K/W, and a case-to-ambient thermal resistance of 8 K/W. The temperature increase of the windings is given by using the formula below; we can substitute Ploss for $I^2 \cdot R$:

$$\Delta T = P_{loss} \cdot (R_{th1} + R_{th2})$$

$$\Delta T = 6{,}22\ \text{W} \cdot (3\ \text{K/W} + 8\ \text{K/W}) = 68{,}4\ \text{K}$$

Because the Kelvin scale uses the same unit increment as the Celsius scale, we can simply substitute the Kelvin value as if it were a Celsius value. If it is assumed that the ambient air temperature is 22°C, then the final temperature of the motor windings can be approximated as:

$$T_{warm} = \Delta T + T_{amb}$$

$$T_{warm} = 68{,}4°\text{C} + 22°\text{C} = 90{,}4°\text{C}$$

Where:

T_{warm} = Winding Temperature

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It is important to be certain that the final temperature of the windings does not exceed the motor's rated value found on the data sheet. In the example given above, the maximum permissible winding temperature is 125°C. Since the calculated winding temperature is only 90,4°C, thermal damage to the motor windings should not be an issue in this application.

One could use similar calculations to answer a different kind of question. For example, an application may require that a motor run at its maximum torque with the hope it will not be damaged by overheating. Suppose it is desired to run the motor at the maximum possible torque with an ambient air temperature of 22°C. The designer wants to know how much torque the motor can safely provide without overheating. Again, the datasheet for the 2668W024CR coreless DC motor specifies a maximum winding temperature of 125°C. So, since the ambient temperature is 22°C, the maximum tolerable rotor temperature increase is: 125°C – 22°C = 103°C

Now we can calculate the increase in coil resistance due to thermal power dissipation:

$$R_{warm} = R \cdot (1 + \alpha_{Cu} \cdot \Delta T)$$

Where:
α_{Cu} = Copper Temperature Coefficient in units of K^{-1} (Inverse Kelvins)

$$R_{warm} = 1{,}03\ \Omega \cdot (1 + 0{,}0039 K^{-1} \cdot 103°C) = 1{,}44\ \Omega$$

So because of coil and magnet heating due to power dissipation from the $I^2 \cdot R$ losses, the coil resistance increased from 1,03 Ω to 1,44 Ω. Now we can recalculate the new torque constant k_M to observe the effects of temperature rise on motor performance:

$$k_{M\text{-}warm} = k_M \cdot (1 + \alpha_M \cdot \Delta T)$$

Where:
α_M = Magnet Temperature Coefficient in units of K^{-1} (Inverse Kelvins)

$$k_{M\text{-}warm} = 28{,}9\ \text{mNm/A} \cdot (1 + (-0{,}0011 K^{-1}) \cdot 103°C) = 25{,}63\ \text{mNm/A}$$

Now we recalculate the new Back-EMF constant k_E and observe the results. From the formula we derived above:

$$k_{E\text{-}warm} = k_{M\text{-}warm} \cdot \left(\frac{2\pi}{60}\right)$$

$$k_{E\text{-}warm} = 25{,}63\ \text{mNm/A} \cdot \left(\frac{2\pi}{60}\right) = 2{,}68\ \text{mV/min}^{-1}$$

$$(k_{E\text{-}cold} = 3{,}02\ \text{mV/min}^{-1})$$

As we can see, the torque constant weakens as a result of temperature increase as does the Back-EMF constant! So the motor's coil resistance, the torque constant, and Back-EMF constant are all negatively impacted for the very simple reason that they are functions of temperature.

We could continue calculating additional parameters as a result of the hotter coil and magnet, but the best results are yielded by performing multiple iterations which is best done by quantitative software. As the motor temperature continues to rise each of the three parameters will change in a fashion which degrades motor performance and increases the power losses. Under continuous operation, a motor may even reach a point of "thermal runaway" which could potentially render the motor damaged beyond repair. This can happen even if initial calculations demonstrated an acceptable temperature rise (using values of R and k_M at ambient temperature).

Note that the maximum allowable current through the motor windings could be increased by decreasing the thermal resistance of the motor. The rotor-to-case thermal resistance R_{th1} is primarily fixed by the motor design. The case-to-ambient thermal resistance R_{th2} can be decreased significantly by the addition of heat sinks. Motor thermal resistances for small DC motors are usually specified with the motor suspended in free air. Therefore, there is usually some heat sinking which results from simply mounting the motor into a thermally conductive framework or chassis. Some manufacturers of larger DC motors specify thermal resistance with the motor mounted into a metal plate of known dimensions and material.

For more information regarding DC coreless brush motor calculations, and how electric motor performance may be affected by thermal power dissipation, please contact a qualified FAULHABER applications engineer. We're always here to help.

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